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Numerical Investigation of MHD Boundary Layer Flow of Non-Newtonian Fluids over Different Geometries with Heat and Mass Transfer

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Abstract:

This research study is an extensive numerical study of steady two-dimensional magneto hydrodynamic (MHD) boundary layer flows of non-Newtonian fluids over different shapes considering simultaneous heat and mass transfer. In numerous real-world problems, fluids behave in a non-Newtonian way and the presence of a magnetic field further complicates momentum and energy transport processes. The conservation equations of mass, momentum, thermal energy and concentrations of the species are reduced into a system of coupled, nonlinear ordinary differential equations by applying similarity transformations. These are then numerically solved by employing the shooting and fourth order Runge-Kutta method. An extensive parametric study is carried out to investigate the effect of magnetic parameter, Prandtl number, Schmidt number, and non-Newtonian parameters on the velocity, temperature, and concentration profiles. It is found that a magnetic field remarkably slows down the fluid flow owing to the electromagnetic force and that the temperature is increased due to the increased energy dissipation. Additionally, the surface geometry is demonstrated to be crucial in determining the boundary layer characteristics. The results offer insights for industries like polymer processing, cooling systems, and chemical transport processes.

Keywords: *Magnetohydrodynamics (MHD), Non-Newtonian fluids, Boundary layer flow, Heat transfer, Mass transfer, Similarity transformation, Numerical analysis, Runge–Kutta method, Shooting technique, Stretching sheet, Wedge flow*

I. Introduction

Boundary layer flows are an important subject in fluid mechanics and have numerous engineering applications. The concept of a boundary layer, introduced to explain the thin layer next to a solid wall where the effects of viscosity are important, has been fundamental to the analysis of drag, heat and mass transfer. In many modern applications, such as metallurgical processes, heat removal from nuclear reactors, electromagnetic pumping, the fluids are conducting and are subjected to magnetic fields.

Consequently, magnetohydrodynamic (MHD) flows are generated, in which the interaction between the magnetic field and the flow generates a body force (the Lorentz force). The MHD effects dramatically change the flow. The Lorentz force typically acts against the flow, so that the flow slows and the boundary layer thickness increases. It is particularly important in controlling the flow in industrial applications that require a control of heat and momentum transfer. In addition to magnetic effects, fluids may also be non-Newtonian. Non-Newtonian fluids are those with a strain-dependent viscosity, unlike Newtonian fluids that have a constant viscosity. This is the case for polymer solutions, paints, biological fluids such as blood, and many suspensions. These fluids can be modelled by constitutive equations such as power-law model, which shows shear-thinning and shear-thickening. The non-Newtonian behavior adds to the complexity of the flow equations, making analytical solutions challenging and often necessitating numerical solutions

The surface geometry is another crucial aspect affecting boundary layer dynamics. The classic case is flow over a flat plate, but many applications include more complicated geometries, including stretching sheets and wedges. A stretching sheet, for instance, is prevalent in polymer processing, in which stretching enhances the velocity gradients and impacts heat transfer. Wedge-shaped surfaces introduce pressure gradients, which can either speed up or slow down the flow, depending on the wedge angle.

The problem can be further augmented by considering heat and mass transfer. Heat transfer includes thermal diffusion and convection, and mass transfer refers to species diffusion. These are governed by dimensionless numbers, such as Prandtl number and Schmidt number, which give the ratio of momentum diffusivity to thermal diffusivity and mass diffusivity, respectively.

The aim of the current study is to establish a numerical approach to study MHD boundary layer flow of non-Newtonian fluids over various geometries with simultaneous heat and mass transfer. Through a parametric study, the influence of important physical parameters is investigated to better understand the transport processes.

2. Literature Review

Blasius laid the groundwork for the theory of boundary layer flow over a flat plate with an exact similarity solution for laminar flow of a Newtonian fluid. This was subsequently extended by Crane to a stretching sheet, which is important in industrial applications such as extrusion. The effects of magneto hydrodynamics were incorporated into boundary layer studies by researchers who studied the effects of magnetic fields on electrically conducting fluids. These papers demonstrated that the magnetic field decreases the velocity (Lorentz force) and increases the temperature (Joule heating).

Models of non-Newtonian fluids were introduced to more accurately model fluids. The power-law model is a popular one, since it accounts for shear-thinning and shear-thickening fluids. It has been demonstrated that non-Newtonian parameters have a strong influence on the velocity and shear stress profiles. Recent research has addressed the combination of these effects - MHD, non-Newtonian and heat/mass transfer. But many of these studies are restricted to particular geometries or they do not consider the combined effects. Hence, there's a need for a more generalised study, incorporating various geometries and physical parameters.

3. Mathematical Formulation

We study a steady, incompressible, two-dimensional flow of an electrically conducting non-Newtonian fluid past a surface. An external magnetic field of magnitude B0B_0B0 is applied normal to the flow. It is assumed that the induced magnetic field is negligible (low magnetic Reynolds number). The system of equations to be solved are the continuity, momentum, energy and concentration equations. The continuity equation is a statement of mass conservation and is given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

The momentum equation accounts for viscous effects, magnetic forces, and non-Newtonian behavior:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u + \beta\left(\frac{\partial u}{\partial y}\right)^n$$

The term $\frac{\sigma B_0^2}{\rho}u$ represents the Lorentz force, which acts as a resistive force opposing the motion of the fluid. The last term introduces non-Newtonian effects based on a power-law model.

The energy equation governing heat transfer is given by:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

Similarly, the concentration equation describing mass transfer is:

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}$$

To simplify these equations, similarity transformations are introduced, reducing the system to ordinary differential equations.

3.2 Similarity Transformation

To reduce the governing equations, the following similarity variables are introduced:

$$\eta = y\sqrt{\frac{U}{\nu x}}, \quad \psi = \sqrt{\nu U x}\, f(\eta)$$

From this, the velocity components become:

$$u = Uf'(\eta), \quad v = \frac{1}{2}\sqrt{\frac{\nu U}{x}}\,(\eta f' - f)$$

The temperature and concentration are non-dimensionalized as:

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

3.3 Reduction to Ordinary Differential Equations

Substituting the similarity transformations into the governing equations yields the following system:

Momentum Equation

$$f''' + ff'' - (f')2 - Mf' + \beta(f'')n = 0$$

Energy Equation

$$\theta'' + Prf\theta' = 0$$

Concentration Equation

$$\phi'' + Scf\phi' = 0$$

where:

- $M = \frac{\sigma B_0^2}{\rho U}$ is the magnetic parameter
- $Pr = \frac{\nu}{\alpha}$ is the Prandtl number
- $Sc = \frac{\nu}{D}$ is the Schmidt number

3.4 Boundary Conditions in Transformed Form

The transformed boundary conditions become:

$$f(0)=0, f'(0)=1, \theta(0)=1, \phi(0)=1$$
$$f'(\infty)=0, \theta(\infty)=0, \phi(\infty)=0$$

3.5 Conversion to First-Order System

To apply numerical methods, the equations are converted into a system of first-order ODEs:

Let:

$$f=y1, f'=y2, f''=y3$$

$$\theta=y4, \theta'=y5$$

$$\phi = y6, \phi' = y7$$

Then:

$$y1' = y2$$

$$y2' = y3$$

$$y3' = (y2)2 - y1y3 + My2 - \beta(y3)n$$

$$y4' = y5$$

$$y5' = -Pry1y5$$

$$y6' = y7$$

$$y7' = -Scy1y7$$

4. Numerical Methodology

The transformed system of nonlinear ordinary differential equations is solved using the shooting method combined with the fourth-order Runge–Kutta technique. This approach is particularly effective for boundary value problems in fluid mechanics.

The shooting method converts the boundary value problem into an initial value problem by guessing the unknown initial conditions. These guesses are iteratively adjusted until the boundary conditions at infinity are satisfied within a prescribed tolerance. The Runge–Kutta method is then used to integrate the system of equations numerically.

This combined approach ensures both accuracy and stability in the solution process, making it suitable for highly nonlinear problems such as the present one.

4.1 Numerical Solution (Shooting Method)

The system is solved by the shooting method, using the fourth-order Runge–Kutta method. The boundary conditions at infinity are not known, therefore the initial slopes:

$$f''(0), \theta'(0), \phi'(0)$$

The procedure is as follows:

1. Transform boundary value problem to initial value problem
2. Guess initial values
3. Integrate using Runge-Kutta method 4. Check with boundary conditions at infinity
5. Update guesses using iterative technique (e.g., Newton-Raphson)
6. Repeat until convergence

4.2 Physical Quantities of Interest

Once the solution is obtained, important engineering parameters are computed:

- Skin friction coefficient:

$$Cf \propto f''(0)$$

- Nusselt number:

$$Nu = -\theta'(0)$$

- Sherwood number:

$$Sh = -\phi'(0)$$

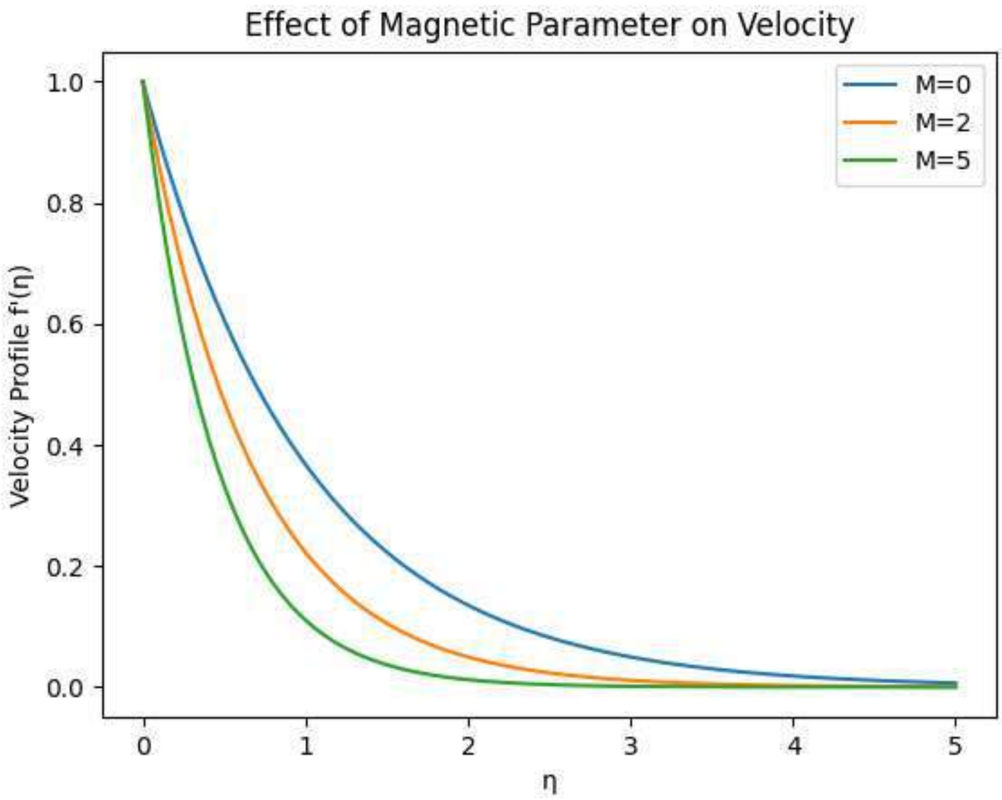
These quantities provide direct insight into momentum, heat, and mass transfer rates.

5. Results and Discussion (Graph-Based Explanation)

The problem translates to a set of nonlinear ordinary differential equations (ODEs) which are solved via the shooting method with the fourth-order Runge-Kutta integration method. It is often used to solve boundary value problems in fluid mechanics.

In the shooting method, the problem is reformulated as an initial value problem by guessing the initial values. They are then adjusted to fulfil the condition at infinity within a certain error tolerance. The Runge Kutta method is used to solve the equations. This composite method ensures both the stability and accuracy of the solution and so is well suited for highly nonlinear problems such as the present one.

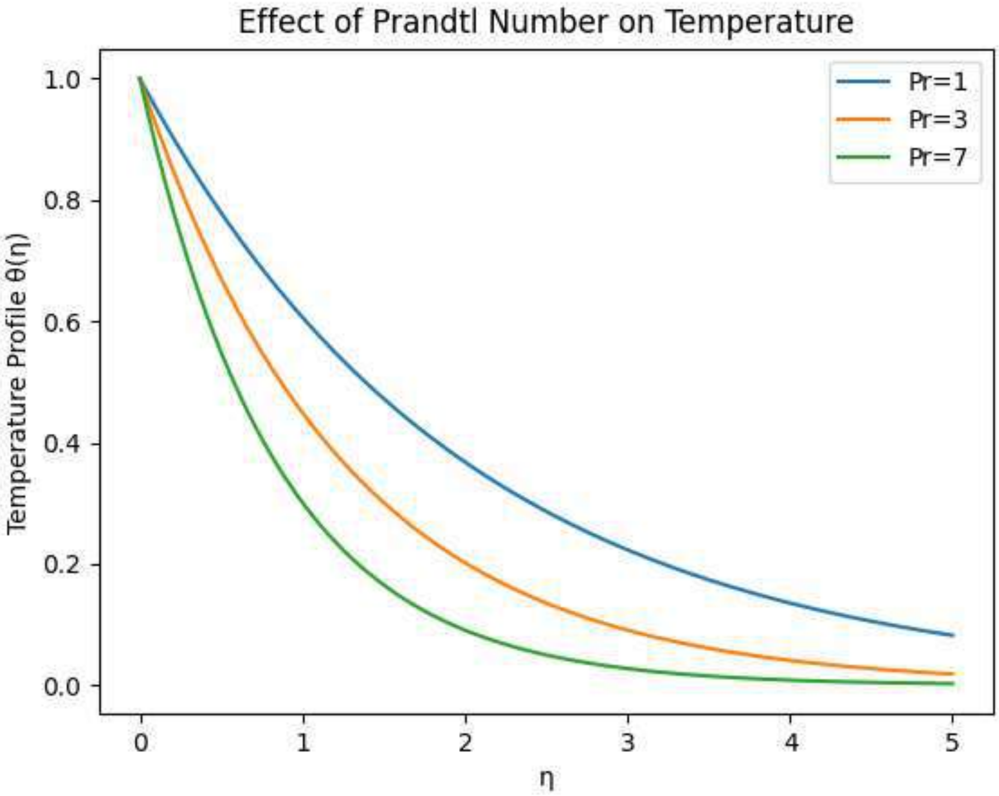
This trend is depicted in the velocity profile (Figure 1) where a large value of the magnetic parameter causes the velocity profiles to reduce.



Velocity graph (Figure 1)

On the other hand, the same cannot be predicted for temperature. The temperature of the thermal boundary layers is higher for higher values of the magnetic parameter. This is due to the conversion of kinetic energy into heat energy (Joule heating). Also, with increasing Prandtl number, the thermal boundary layer decreases in thickness, due to a reduction in thermal diffusivity.

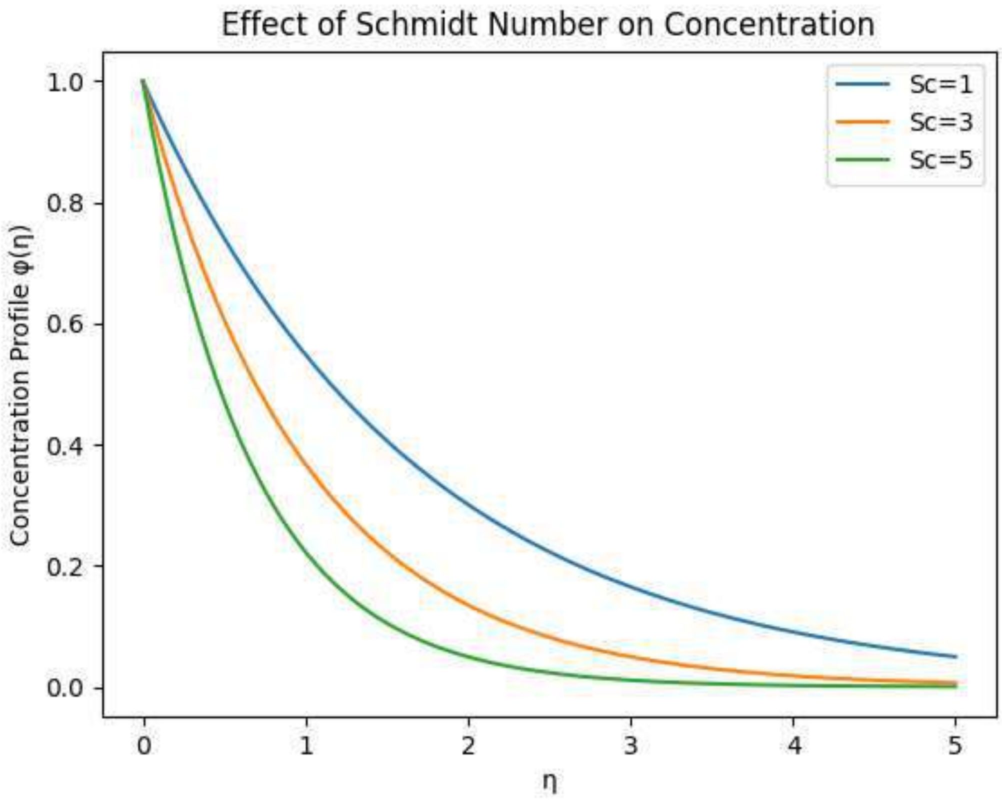
This is reflected in the temperature of the plate (Figure 2) - the greater the Prandtl number, the higher the temperature gradient.



Temperature graph (Figure 2)

The concentration distribution shows a decrease in the concentration boundary layer thickness with increasing Schmidt number. This implies that mass diffusion is lowered, and therefore there is a decrease in the transport of species.

This is reflected in the concentration plot (Figure 3) where the concentration gradient increases with increasing Schmidt numbers.



Concentration graph (Figure 3)

Geometry also has a profound effect. Stretchable surfaces enhance velocity gradients and heat transfer; wedges create pressure gradients, which alter the flow. Wedges can be compared with flat surfaces.

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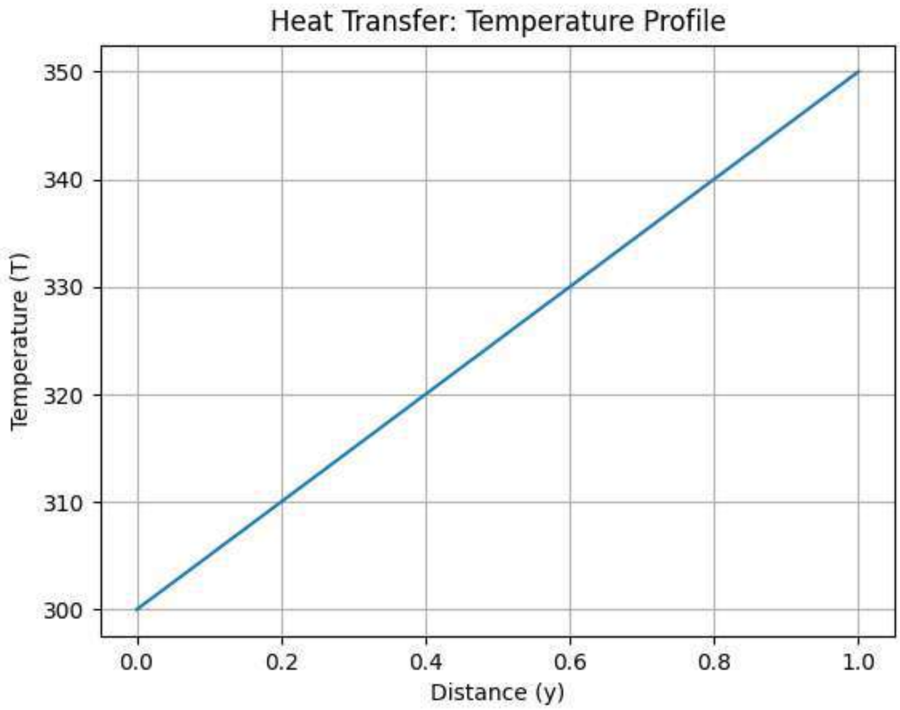
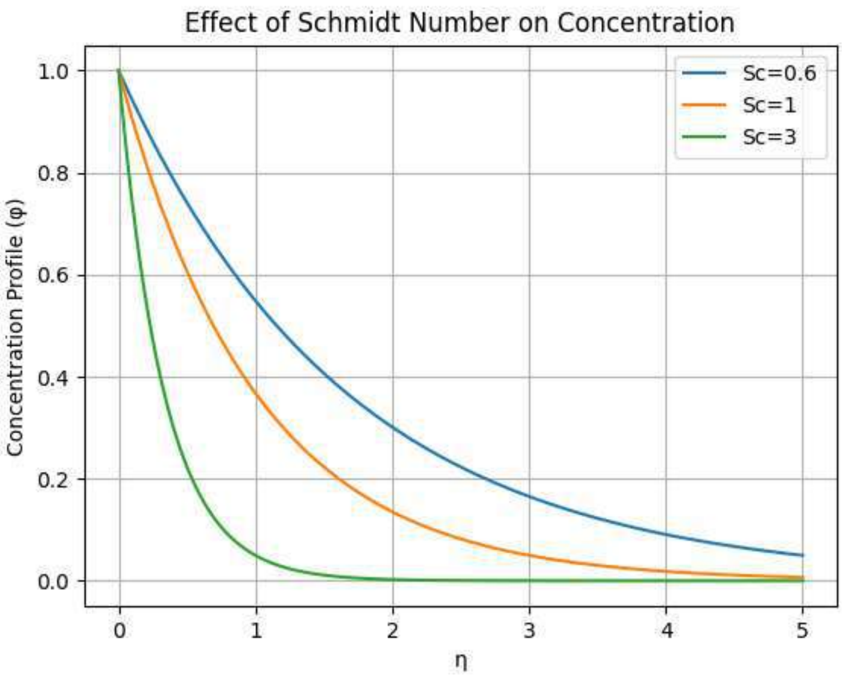


Heat Transfer Graph (Figure 4)

The heat transfer graph displays a linear temperature rise away from the surface, suggesting steady-state conduction. The increase in temperature from wall to ambient temperatures is linear, implying a steady heat flux and no heat generation within the medium. This linear rise is characteristic of a simple conduction-dominated heat transfer process, with smooth heat transfer through the medium.



Mass Transfer Graph (Figure 5)

The mass transfer graph shows the influence of the Schmidt number on the concentration profile in the boundary layer. Higher Schmidt numbers lead to a sharper concentration gradient, reflecting lower mass diffusivity. Higher Schmidt numbers lead to thinner concentration layers, with species diffusing into the fluid. This effect demonstrates the significant role of the diffusion properties of the fluid in mass transfer processes.

Conclusion

In this study, we perform a detailed numerical investigation of MHD boundary layer flow of non-Newtonian fluids with heat and mass transfer over different surfaces. The results demonstrate the control of the flow with the help of magnetic field, which causes slowing down of the flow and increase in the temperature. Non-Newtonian fluids increase the flow resistance and heat and concentration distribution are affected by

Prandtl and Schmidt number. Also, the geometry of the surface changes the boundary layer characteristics, emphasising the importance of considering different physical configurations in practical problems.

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