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# A Multi-Attribute Decision Making Model Based on Complex Fermatean Fuzzy Hypersoft set

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**Abstract:** *Complex Fermateen fuzzy soft set(CFFSS) deal with the parameters, but not with the sub-parameters. So to discuss sub-parameters , this paper will be used to find a way to be more accurate in the selection of anything of our need by thier parameters and sub-parametrs. So this research paper will extend CFFSS to the complex Fermatean fuzzy hyper-soft set(CFFHSS), because in hypersoft not only parameters but also sub-parameters are discussed and also CFFHSS is more flexible for solving complex uncertainties. This research paper will be use to find out the solution for the selection of a suitable Mobile Phone for a middle class boy, because now a days it is a big problem for middle class boys to find a suitable Moble phone for their use. A more enlightened decision-making process is made possible by the efficient evaluation of several criteria and the related subcriteria that can be achieved by utilizing the CFFHSS framework.*

**Keywords:** *Multi-Attribute Decision Making, complex Fermatean fuzzy hypersoft set(CFFHSS), Mobile Phones selection*

## 1 Introduction

In this Decade the major issue of today's generation is selection of Mobile phone, because of wide uses of moble phones i.e Internet suffering, videos and photos. For selecting a mobile phone mostly watched things are its features i.e camera, storage, mobile network, and its battery. Some companies offers good camera quality but not other features and some companies offers good battery and storage but not others and vice versa. For selecting a low budget good mobile phone a person cannot have a mobile with balance features. This selection process may needs keen observation and analysis of the factors associated with the selection element, but it may involve uncertainty due to incomplete or imprecise information.

For this, the first concept of Fuzzy set was introduced by Zedah [1]. It deals with uncertainty by tagging each element with a real value between 0 and 1. Then for a non-membership values, Atanassov [2] extended fuzzy set to intuitionistic fuzzy set (IFS). Two values one for membership and the other for non membership was introduced for each element in this set. These values should be between 0 and 1, and the addition of these two values must be within this range. Nevertheless, the sum of the values of the other membership and non-membership function options does not fall between 0 and 1. Membership value is for the belongingness and non-membership was for the not belongingness. Pythagorean fuzzy set, for calculating the vagueness while accounting for membership grades, was extended by Yager and Abbasov [3].

The Pythagorean structure was involved with Square of membership value and non-membership value, which should be lie between 0 and 1. And therefor this structure can not handle wide arrangements of numbers so, Senapati and Yager[4] developed the idea of Pythagorean fuzzy set by proposing the notion of Fermatean FS (FFS). When the sum of the cubes of membership and non-membership functions does not fall inside [0, 1], FFS requires that it does. The similarity metric for FS's hybrid set structure was developed by Zhu et al.[5]. All above mentioned structures deal with real numbers which give us only real values So, In order to extend the concept of fuzzy set from real numbers to complex numbers, Ramot et al. [6] proposed the complex fuzzy set (cFS). Every element of complex fuzzy set is attatched to a complex-valued function that is tagged to phase term value and amplitude terms and has values inside the unit circle.

Garg et al. [7] gives the idea of intitiunistic fuzzy set(IFS) to complex IFS (CIFS) by replacing real-valued membership and non-membership values with complex-valued ones. Molodtsov[8] gives the idea of soft set (SS) to deal with uncertainty in parametric way. This theory was a generalization of FS theory and provides an adaptable framework for handling imprecise information. A very popular extension in decision-making problems is fuzzy soft set (FSS), which was developed by Cagman et al. [9] as a generalized wprk of Molodtsov. Xu et al. [10] developed intuitionistic fuzzy soft sets (IFSS), which explore the idea of IFSs into the SS structure and gives a brief the toolkit for handling uncertain data. By making the condition Ref. [11] presents the idea of Pythagorean FSS (PyFSS), which calculates the vaguness in the decision-making process and is a component of IFSS. Compared to IFSS, PyFSS has a broader framework. In [12], developers gives the idea of the Fermatean FSS (FFSS). The extension of complex fuzzy set (FS) theory marks an important step forward in dealing with uncertainty .

Alkouri and Salleh [13] gives the definition of complex intuitionistic fuzzy sets (IFS). The extension of complex IFSS was given by Maji et al. [14] as an advanced extension of IFSs and SSs. The idea of comlex farmateen fuzzy soft set was given by Asghar A, Khan A. [15]. Samarchande [16] extends the idea of soft set to Hypersoft set to discuss the sub parameters of parameters and to be more accurate in uncertainty. yolcu and ozturk [17] covert the HyperSoft set to fuzzy HyperSoft set(FHSS). Yolcu A, Smarandache F [19] Introduced the idea of Intiutionistic fuzzy hyper soft set (IFHSS). Siddique I, Zulqarnain RM [18] gives the idea of

Pythagorean fuzzy hyper soft set (PFHSS).Kirisci M. [20] developd the idea of Farmatean fuzzy hyper soft set (FFHSS). Ahsan M, Saeed M, Mehmood A [21] extends the Fuzzy hyper soft sets to Complex Fuzzy Hypersoft set (CFHS). Rahman AU, Saeed M, Smarandache F[22] introduces the Complex Intiutionistic fuzzy hyper soft set (CIFHSS).

## 1.1 Article's Arrangement

For desired results, this paper is consist on different sections. In Section [2] a comprehensive definition of Complex farmatean fuzzy hypersoft set is given and further some new formula's for similarity measure are discussed for this structure such as Hamming similarity measure and Euclidean similarity measure etc. In Section 3 A comprehensive algorithm is proposed for solving any problem with uncertainty and ambiguities and for this a case study is discussed to prove that our proposed algorithm is fruitfull, and also sensitivity analysis is discussed by calculating some statistical tools i.e Mean, Geometric Mean, and Harmonic Mean etc. In Section[5] research paper is concluded by giving final arguments.

# 2 Characteristics of Complex Farmatean Fuzzy Hypersoft Set

In This section some basic definition will be recall from the literature which are important to understand the main concept of the paper, and then the main definition of complex fermatean fuzzy hypersoft set will be presented.

## 2.1 Preliminaries

Let's recall some basic definitions to understand the main proposed definition and its properties.

**Definition 1** *[6] Let* $\mho$ *be all-round set, then a set* $Y$ *is said to be Fuzzy set(FS) over* $\mho$*, is defined as:*

$$Y = \{(\upsilon, \nu_s(\upsilon)) : \upsilon \in \mho, \nu_s(\upsilon) \in [0,1]\}$$

**Definition 2** *[16] Let* $\mho$ *be a all-round set and* $\varrho = \{\varrho_1, \varrho_2, \ldots. \varrho_n\}$ *be a set of attributes with* $\varepsilon_1, \varepsilon_2 \ldots. \varepsilon_n$ *be sub attributes to each* $\varrho_n$ *such that* $\varepsilon_k \cap \varepsilon_l = \{\}, \forall k, l \in \{1,2,3,4 \ldots. n\}$ *for* $k \neq l$*. Consider* $\varepsilon_1 \times \varepsilon_2 \times \varepsilon_3 \ldots. \times \varepsilon_n = \beta$ *be collection of multi-attributes, then a pair* $(F, \beta)$ *is said Hypersoft set , where* $F: \varepsilon_1 \times \varepsilon_2 \times \varepsilon_3 \ldots. \times \varepsilon_n \to P(\mho)$.

**Definition 3** *[18] Let* $\mho$ *be a all-round set and* $\varrho = \{\varrho_1, \varrho_2, \ldots. \varrho_n\}$ *be a set of attributes with* $\varepsilon_1, \varepsilon_2 \ldots. \varepsilon_n$ *be sub attributes to each* $\varrho_n$ *such that* $\varepsilon_k \cap \varepsilon_l = \{\}, \forall k, l \in \{1,2,3,4 \ldots. n\}$ *for* $k \neq l$*. Consider* $\varepsilon_1 \times \varepsilon_2 \times \varepsilon_3 \ldots. \times \varepsilon_n = \beta$ *be collection of multi-attributes, and* $PFS(\mho)$ *be collection of Pythagorean fuzzy subset of* $\mho$*, then* $F_\beta$ *is said to be Pythagorean fuzzy hyper soft set (PFHSS) over* $\mho$ *and defined as:*

$$F_\beta = \{(\widetilde{M}, F_\beta(\widetilde{M})) \mid \widetilde{M} \in \beta,\ F_\beta(\widetilde{M}) \in \mathrm{PFS}(\mho)\},$$

where $F_\beta(\widetilde{M}) = \{(\upsilon, \mu_{F(\widetilde{M})}(\upsilon), \nu_{F(\widetilde{M})}(\upsilon)), \upsilon \epsilon \mho\}$ with $\mu_{F(\widetilde{M})}(\upsilon), \nu_{F(\widetilde{M})}(\upsilon)$ are membership and non-membership function with conditions $0 \leq \mu^2_{F(\widetilde{M})}(\upsilon) + \nu^2_{F(\widetilde{M})}(\upsilon) \leq 1$ where

$$\mu_{F(\widetilde{M})}(\upsilon), \nu_{F(\widetilde{M})}(\upsilon): \mho \to [0,1].$$

**Definition 4** *[22] Let $\mho$ be a all-round set and $\varrho = \{\varrho_1, \varrho_2, \ldots . \varrho_n\}$ be a set of attributes with $\varepsilon_1, \varepsilon_2 \ldots . \varepsilon_n$ be sub attributes to each $\varrho_n$ such that $\varepsilon_k \cap \varepsilon_l = \{\}, \forall k, l \in \{1,2,3,4 \ldots . n\}$ for $k \neq l$. Consider $\varepsilon_1 \times \varepsilon_2 \times \varepsilon_3 \ldots . \times \varepsilon_n = \beta$ be collection of multi-attributes, and $CIFS(\mho)$ be collection of Complex Intiutionistic fuzzy subset of $\mho$, then $F_\beta$ is said to be Complex Intiutionistic fuzzy hyper soft set (CIFHSS) over $\mho$ and defined as:*

$$F_\beta = \{(\widetilde{M}, F_\beta(\widetilde{M})) \mid \widetilde{M} \in \beta,\ F_\beta(\widetilde{M}) \in \mathrm{CIFS}(\mho)\},$$

where $F_\beta(\widetilde{M}) = \{(\upsilon, \mu_{F(\widetilde{M})}(\upsilon), \nu_{F(\widetilde{M})}(\upsilon)), \upsilon \epsilon \mho\}$ with $\mu_{F(\widetilde{M})}(\upsilon), \nu_{F(\widetilde{M})}(\upsilon)$ are membership and non-membership functions Where $\mu_{F(\widetilde{M})}(\upsilon) = r_{\Theta F} e^{i2\pi\vartheta_{\Theta F}}$ and $\nu_{F(\widetilde{M})}(\upsilon) = r_{\Xi F} e^{i2\pi\vartheta_{\Xi F}}$ with conditions $0 < r_{\Theta F} + r_{\Xi F} < 1$ and $0 < \vartheta_{\Theta F} + \vartheta_{\Xi F} < 1$. and $r_{\Theta F}, \vartheta_{\Theta F}: \mho \to [0,1]$.

## 2.2 Complex Fermatean Fuzzy Hypersoft Set

**Definition 5** *Let $\mho$ be a all-round set and $\varrho = \{\varrho_1, \varrho_2, \ldots . \varrho_n\}$ be a set of attributes with $\varepsilon_1, \varepsilon_2 \ldots . \varepsilon_n$ be sub attributes to each $\varrho_n$ such that $\varepsilon_k \cap \varepsilon_l = \{\}, \forall k, l \in \{1,2,3,4 \ldots . n\}$ for $k \neq l$. Consider $\varepsilon_1 \times \varepsilon_2 \times \varepsilon_3 \ldots . \times \varepsilon_n = \beta$ be collection of multi-attributes, and $CFFS(\mho)$ be collection of Complex Farmatean fuzzy subset of $\mho$, then $F_\beta$ is said to be Complex Farmatean fuzzy hyper soft set (CFFHSS) over $\mho$ and defined as:*

$$F_\beta = \{(\widetilde{M}, F_\beta(\widetilde{M})) \mid \widetilde{M} \in \beta,\ F_\beta(\widetilde{M}) \in \mathrm{CFFS}(\mho)\},$$

where $F_\beta(\widetilde{M}) = \{(\upsilon, \mu_{F(\widetilde{M})}(\upsilon), \nu_{F(\widetilde{M})}(\upsilon)), \upsilon \epsilon \mho\}$ with $\mu_{F(\widetilde{M})}(\upsilon), \nu_{F(\widetilde{M})}(\upsilon)$ are membership and non-membership functions Where $\mu_{F(\widetilde{M})}(\upsilon) = r_{\Theta F} e^{i2\pi\vartheta_{\Theta F}}$ and $\nu_{F(\widetilde{M})}(\upsilon) = r_{\Xi F} e^{i2\pi\vartheta_{\Xi F}}$ with condition $0 < r^3_{\Theta F} + r^3_{\Xi F} < 1$ and $0 < \vartheta^3_{\Theta F} + \vartheta^3_{\Xi F} < 1$.

**Example 1** *Let $\mho = \{Dell, Lenovo\} = \{\upsilon_1, \upsilon_2\}$ be the all-round set and the collection of attributes is $\varrho = \{Ram, Rom, Processor\} = \{\varrho_1, \varrho_2, \varrho_3\}$. Their corresponding sub-attribute are given as $\varrho_1 = \{4gb, 8gb\} = \{m_{11}, m_{12}\}$, $\varrho_2 = \{128gb, 256gb, 512gb\} = \{m_{21}, m_{22}, m_{23}\}$, $\varrho_3 = \{Intelcorei5, Intelcorei7\} = \{m_{31}, m_{32}\}$. Define $\beta = \varrho_1 \times \varrho_2 \times \varrho_3$, such that,*

$$\beta = \{m_{11}, m_{12}\} \times \{m_{21}, m_{22}, m_{23}\} \times \{m_{31}, m_{32}\} = \{(m_{11}, m_{21}, m_{31}), (m_{11}, m_{21}, m_{32}),$$

$$(m_{11}, m_{22}, m_{31}), (m_{11}, m_{22}, m_{32}), (m_{11}, m_{23}, m_{31}), (m_{11}, m_{23}, m_{32}), (m_{12}, m_{21}, m_{31}),$$

$$(m_{12}, m_{21}, m_{32}), (m_{12}, m_{22}, m_{31}), (m_{12}, m_{22}, m_{32}), (m_{12}, m_{23}, m_{31}), (m_{12}, m_{23}, m_{32})\}$$

$$= \{\widetilde{M}_1, \widetilde{M}_2, 9deM_3, \widetilde{M}_4, \widetilde{M}_5, \widetilde{M}_6, \widetilde{M}_7, \widetilde{M}_8, \widetilde{M}_9, \widetilde{M}_{10}, \widetilde{M}_{11}, \widetilde{M}_{12}\}$$

then CFFHSS over Ʊ is given as:

$$F_\beta = \left\{\begin{array}{l}
(\widetilde{M}_1, (\upsilon_1, 0.81e^{i2\pi(0.57)}, 0.49e^{i2\pi(0.55)})), (\upsilon_2, (0.56e^{i2\pi(0.67)}, 0.78e^{i2\pi(0.47)})),\\
(\widetilde{M}_2, (\upsilon_1, 0.81e^{i2\pi(0.72)}, 0.65e^{i2\pi(0.32)}), (\upsilon_2, (0.42e^{i2\pi(0.56)}, 0.91e^{i2\pi(0.62)})),\\
(\widetilde{M}_3, (\upsilon_1, 0.69e^{i2\pi(0.49)}, 0.58e^{i2\pi(0.59)})), (\upsilon_2, (0.87e^{i2\pi(0.53)}, 0.45e^{i2\pi(0.40)})),\\
(\widetilde{M}_4, (\upsilon_1, 0.75e^{i2\pi(0.54)}, 0.49e^{i2\pi(0.44)})), (\upsilon_2, (0.75e^{i2\pi(0.45)}, 0.62e^{i2\pi(0.64)})),\\
(\widetilde{M}_5, (\upsilon_1, 0.85e^{i2\pi(0.65)}, 0.54e^{i2\pi(0.47)})), (\upsilon_2, (0.54e^{i2\pi(0.65)}, 0.69e^{i2\pi(0.40)})),\\
(\widetilde{M}_6, (\upsilon_1, 0.83e^{i2\pi(0.62)}, 0.49e^{i2\pi(0.44)})), (\upsilon_2, (0.92e^{i2\pi(0.45)}, 0.45e^{i2\pi(0.64)})),\\
(\widetilde{M}_7, (\upsilon_1, 0.81e^{i2\pi(0.90)}, 0.57e^{i2\pi(0.12)})), (\upsilon_2, (0.52e^{i2\pi(0.21)}, 0.83e^{i2\pi(0.82)})),\\
(\widetilde{M}_8, (\upsilon_1, 0.34e^{i2\pi(0.77)}, 0.88e^{i2\pi(0.61)})), (\upsilon_2, (0.33e^{i2\pi(0.31)}, 0.89e^{i2\pi(0.80)})),\\
(\widetilde{M}_9, (\upsilon_1, 0.33e^{i2\pi(0.64)}, 0.76e^{i2\pi(0.42)})), (\upsilon_2, (0.43e^{i2\pi(0.41)}, 0.90e^{i2\pi(0.72)})),\\
(\widetilde{M}_{10}, (\upsilon_1, 0.64e^{i2\pi(0.75)}, 0.73e^{i2\pi(0.40)})), (\upsilon_2, (0.49e^{i2\pi(0.21)}, 0.86e^{i2\pi(0.89)})),\\
(\widetilde{M}_{11}, (\upsilon_1, 0.92e^{i2\pi(0.93)}, 0.65e^{i2\pi(0.11)})), (\upsilon_2, (0.38e^{i2\pi(0.31)}, 0.88e^{i2\pi(0.80)})),\\
(\widetilde{M}_{12}, (\upsilon_1, 0.39e^{i2\pi(0.32)}, 0.90e^{i2\pi(0.84)})), (\upsilon_2, (0.85e^{i2\pi(0.36)}, 0.55e^{i2\pi(0.80)}))
\end{array}\right\}$$

## 2.3 Similarity Measures between two CFFHSS

**Definition 6** *Hamming similarity measurement method measures the average absolute distance between the corresponding elements of two complex Fermateen fuzzy hypersoft sets. Let* $F_\beta = \{(\widetilde{M}, (\upsilon, \mu_{F(\widetilde{m})}(\upsilon) = r_{\Theta F}e^{i2\pi\vartheta_{\Theta F}}, \nu_{F(\widetilde{M})}(\upsilon)) = r_{\Xi F}e^{i2\pi\vartheta_{\Xi F}}, \upsilon \in Ʊ, \widetilde{M} \in \beta\}$ *and* $H_\eta = \{(\widetilde{N}, (\upsilon, \zeta_{F(\widetilde{N})}(\upsilon) = r_{\Theta H}e^{i\vartheta_{\Theta H}}, \gamma_{F(\widetilde{N})}(\upsilon)) = r_{\Xi H}e^{i\vartheta_{\Xi H}}, \upsilon \in Ʊ, \widetilde{N} \in \eta\}$ *be to CFFHSS,then*

$$S_H(F_\beta, H_\eta) = 1 - \frac{1}{2mn}\sum_{j=1}^{m}\sum_{i=1}^{n} (|r^i_{\Theta F}(\upsilon_j) - r^i_{\Theta H}(\upsilon_{j)}|$$

$$+|\vartheta^i_{\Theta F}(\upsilon_j) - \vartheta^i_{\theta F}(\upsilon_j)| + |r^i_{\Xi F}(\upsilon_j) - r^i_{\Xi H}(\upsilon_j)| + |\vartheta^i_{\Xi F}(\upsilon_j) - \vartheta^i_{\Xi H}(\upsilon_j)|)$$

where $\mu_{F(\widetilde{M})}(\upsilon)$ is membership value of F. $\zeta_{F(\widetilde{N})}(\upsilon)$ is membership value of H. $\nu_{F(\widetilde{M})}(\upsilon)$ is non-membership value of F. $\gamma_{F(\widetilde{N})}(\upsilon)$ is non-membership value of H.

**Definition 7** *In Euclidean Similarity Measurements, it follows the same structure as Hamming measurement, but it just takes square of the differences of values rather then absolute differences of the magnitude and phase, In the complex plane, this translates to the Euclidean (geometric) distance, which penalizes greater disparities more severely. The Formula for the Euclidean Similarity measurement between two CFFHSS* $F_\beta = \{(\widetilde{M}, (\upsilon, \mu_{F(\widetilde{M})}(\upsilon) = r_{\Theta F}e^{i2\pi\vartheta_{\Theta F}}, \nu_{F(\widetilde{M})}(\upsilon)) = r_{\Xi F}e^{i2\pi\vartheta_{\Xi F}}, \upsilon \in Ʊ, \widetilde{M} \in \beta\}$ *and*

$H_{\eta} = \{(\widetilde{N}, (\upsilon, \zeta_{F(\widetilde{N})}(\upsilon) = r_{\Theta H} e^{i\vartheta_{\Theta H}}, \gamma_{F(\widetilde{N})}(\upsilon)) = r_{\Xi H} e^{i\vartheta_{\Xi H}}, \upsilon \in \mho, \tilde{n} \in \eta\}$ *can be written as*

$$S_E(F_\beta, H_\eta) = 1 - \frac{1}{2mn} \sum_{j=1}^{m} \sum_{i=1}^{n} [|r_{\Theta F}^{i}(\upsilon_j) - r_{\Theta H}^{i}(\upsilon_{j)}|^2$$

$$+|\vartheta_{\Theta F}^{i}(\upsilon_j) - \vartheta_{\theta F}^{i}(\upsilon_j)|^2 + |r_{\Xi F}^{i}(\upsilon_j) - r_{\Xi H}^{i}(\upsilon_j)|^2 + |\vartheta_{\Xi F}^{i}(\upsilon_j) - \vartheta_{\Xi H}^{i}(\upsilon_j)|^2]$$

**Definition 8** *Another Similarity measurements between two CFFHSS* $F_\beta = \{(\widetilde{M}, (u, \mu_{F(\widetilde{M})}(u) = r_{\Theta F} e^{i2\pi\vartheta_{\Theta F}}, \nu_{F(\widetilde{M})}(\upsilon)) = r_{\Xi F} e^{i2\pi\vartheta_{\Xi F}}, \upsilon \in \mho, \widetilde{M} \in \beta\}$ *and* $H_{\eta} = \{(\widetilde{N}, (\upsilon, \zeta_{F(\widetilde{N})}(\upsilon) = r_{\Theta H} e^{i\vartheta_{\Theta H}}, \gamma_{F(\widetilde{N})}(\upsilon)) = r_{\Xi H} e^{i\vartheta_{\Xi H}}, \upsilon \in \mho, \widetilde{N} \in \eta\}$ *can be defined as*

$$S_V(F_\beta, H_\eta) = 1 - \frac{1}{2mn} \sum_{j=1}^{m} \sum_{i=1}^{n} [|r_{\Theta F}^{i}(\upsilon_j) - r_{\Theta H}^{i}(\upsilon_{j)}|^3$$

$$+|\vartheta_{\Theta F}^{i}(\upsilon_j) - \vartheta_{\theta F}^{i}(\upsilon_j)|^3 + |r_{\Xi F}^{i}(\upsilon_j) - r_{\Xi H}^{i}(\upsilon_j)|^3 + |\vartheta_{\Xi F}^{i}(\upsilon_j) - \vartheta_{\Xi H}^{i}(\upsilon_j)|^3]$$

## 3 Proposed Algorithm

The proposed Multi-attribute decision making technique based on the complex fermatean fuzzy soft set has the following steps:

1. Design the CFFHSS based on set of alternatives $\mho = \{\upsilon_1, \upsilon_2, \upsilon_3, \dots, \upsilon_n\}$ by using set of key parameters $P = \{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_m\}$ and sub-parameters $\beta = \{\widetilde{M}_1, \widetilde{M}_2, \widetilde{M}_3, \widetilde{M}_4, \dots, \widetilde{M}_m\}$ with the help of experts' opinions, e.g.

$$F(\widetilde{M}_1) = \left\{\langle r_{\Theta F}^{1}(\beta) e^{i2\pi\vartheta_{\Theta F}^{1}(\beta)}, r_{\Xi F}^{1}(\beta) e^{i2\pi\vartheta_{\Xi F}^{1}(\beta)}\rangle\right\}$$

$$F(\widetilde{M}_2) = \left\{\langle r_{\Theta F}^{2}(\beta) e^{i2\pi\vartheta_{\Theta F}^{2}(\beta)}, r_{\Xi F}^{2}(\beta) e^{i2\pi\vartheta_{\Xi F}^{2}(\beta)}\rangle\right\}$$

$$\vdots$$

$$F(\widetilde{M}_m) = \left\{\langle r_{\Theta F}^{m}(\beta) e^{i2\pi\vartheta_{\Theta F}^{m}(\beta)}, r_{\Xi F}^{m}(\beta) e^{i2\pi\vartheta_{\Xi F}^{m}(\beta)}\rangle\right\}$$

2. Rationalize the CFFHSS decision sets for each alternative $\upsilon_i$ as given in Table

| CFFHSS DV | $\upsilon_i$ |
|---|---|
| $\widetilde{M}_1$ | $\begin{pmatrix} r_{\Theta F}^{1}(\beta)e^{i2\pi\vartheta_{\Theta F}^{1}(\beta)} \\ r_{\Xi F}^{1}(\beta)e^{i2\pi\vartheta_{\Xi F}^{1}(\beta)} \end{pmatrix}$ |
| | |
| $\widetilde{M}_2$ | $\begin{pmatrix} r_{\Theta F}^{2}(\beta)e^{i2\pi\vartheta_{\Theta F}^{2}(\beta)} \\ r_{\Xi F}^{2}(\beta)e^{i2\pi\vartheta_{\Xi F}^{2}(\beta)} \end{pmatrix}$ |
| | |
| $\vdots$ | $\vdots$ |
| | |
| $\widetilde{M}_m$ | $\begin{pmatrix} r_{\Theta F}^{m}(\beta)e^{i2\pi\vartheta_{\Theta F}^{m}(\beta)} \\ r_{\Xi F}^{m}(\beta)e^{i2\pi\vartheta_{\Xi F}^{m}(\beta)} \end{pmatrix}$ |

Table 1: Rationalized CFFHSS Decision Values

3. Create the Ideal CFFHSS by using set of sub-parameters

$$\beta = \{\widetilde{M}_1, \widetilde{M}_2, \widetilde{M}_3, \widetilde{M}_4, \dots, \widetilde{M}_m\}$$

from step 1, for ideal alternatives $u$ as given in table 2.

| CFFHSS DV | $\upsilon$ |
|---|---|
| $\widetilde{M}_1$ | $\begin{pmatrix} r_{\Theta F}^{1}(\beta)e^{i2\pi\vartheta_{\Theta F}^{1}(\beta)} \\ r_{\Xi F}^{1}(\beta)e^{i2\pi\vartheta_{\Xi F}^{1}(\beta)} \end{pmatrix}$ |
| | |
| $\widetilde{M}_2$ | $\begin{pmatrix} r_{\Theta F}^{2}(\beta)e^{i2\pi\vartheta_{\Theta F}^{2}(\beta)} \\ r_{\Xi F}^{2}(\beta)e^{i2\pi\vartheta_{\Xi F}^{2}(\beta)} \end{pmatrix}$ |
| | |
| $\vdots$ | $\vdots$ |
| | |
| $\widetilde{M}_m$ | $\begin{pmatrix} r_{\Theta F}^{m}(\beta)e^{i2\pi\vartheta_{\Theta F}^{m}(\beta)} \\ r_{\Xi F}^{m}(\beta)e^{i2\pi\vartheta_{\Xi F}^{m}(\beta)} \end{pmatrix}$ |

Table 2: Ideal Rationalized CFFHSS Decision Values

4. Find Similarity measures $S_H(\upsilon, \upsilon_i)$, $S_E(\upsilon, \upsilon_i)$ and $S_V(\upsilon, \upsilon_i)$ by using definitions (6), (7) and (8).

5. Calculate the mean of similarity measures i.e $\frac{S_H+S_E+S_V}{3}$.

6. Rank the similarity measure from low to high value.

7. Select the option with the highest similarity measure that provides the best alternative among the available choices.

After completing all these steps, the Algorithm offers guidance for organizing the evaluation and comparison of various sites based on a range of key sub-parameters in order to select the best sites for warehouse of companies that align with their interests and aspirations. Furthermore, utilizing the formula for the CFFHSS similarity measure enhances decision-making accuracy and clarity, leading to best possible outcomes.

## 3.1 Case Study

In this Modern era, the major problem of today's youth is that they should have the best and cheapest Mobile phones. Mobile phones plays a vital role now a days because of its versatile uses. i.e for internet browsing a mobile phone should have latest network option availible i.e 4G or 5G. and for the selfies and photos camera of mobile should be of good quality. So therefore selection of mobile phone is also a very important task. Let a boy X wants a mobile phone for his uses i.e whatsapp, facebook, selfies and for playing games. So therefore he wants a mobile which have high quality camera, good storage and ram, and for long use he also wants a healthy battery. In selection of mobile the major problem is that different companies offers a lot of mobile phones which are actually of good quality, but major problem is some companies have good camera but not battery or ram and some companies offers good battery health or ram but not camera and vice versa. So to handle this issue boy choose some companies on the basis of his price range which offers all things according to his choice. i.e $\upsilon_1 = Samsung\ mobile$, $\upsilon_2 = Tecno\ moble$ , $\upsilon_3 = Infinix\ mobile$ , $\upsilon_4 = oppo\ mobile$. Boy choose all these mobile companies on the basis of some parameters i.e $\lambda_1 = RAM$ , $\lambda_2 = mainCamera$ , $\lambda_3 = battery$. For the selection of best mobile himself, he further divided the parameters into subparameters i.e

$$\lambda_1 = RAM = \{4gb, 6gb\} = \{m_{11}, m_{12}\}$$

$$\lambda_2 = mainCamera = \{32mp\} = \{m_{21}\}$$

$$\lambda_3 = battery = \{4500mhz, 5000mhz\} = \{m_{31}, m_{32}\}.$$

So now sub-parameter is

$$\beta = \lambda_1 \times \lambda_2 \times \lambda_3 = \begin{Bmatrix} (m_{11}, m_{21}, m_{31}), (m_{11}, m_{21}, m_{32}), \\ (m_{12}, m_{21}, m_{31}), (m_{12}, m_{21}, m_{32}) \end{Bmatrix}$$

i.e

$$\beta = \{\widetilde{M}_1, \widetilde{M}_2, \widetilde{M}_3, \widetilde{M}_4\}$$

1. The CFFHSS for the set of alternatives $\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4$ by using set of sub-parameters $\beta = \{\widetilde{M}_1, \widetilde{M}_2, \widetilde{M}_3, \widetilde{M}_4\}$ will be given as

$$F(\widetilde{M}_1) = \begin{Bmatrix} (\upsilon_1, \langle 0.70e^{2i\pi(0.44)}, 0.79e^{2i\pi(0.67)} \rangle) \\ (\upsilon_2, \langle 0.81e^{2i\pi(0.75)}, 0.66e^{2i\pi(0.29)} \rangle) \\ (\upsilon_3, \langle 0.47e^{2i\pi(0.65)}, 0.92e^{2i\pi(0.30)} \rangle) \\ (\upsilon_4, \langle 0.85e^{2i\pi(0.46)}, 0.68e^{2i\pi(0.67)} \rangle) \end{Bmatrix}$$

$$F(\widetilde{M}_2) = \begin{Bmatrix} (\upsilon_1, \langle 0.65e^{2i\pi(0.69)}, 0.81e^{2i\pi(0.42)} \rangle) \\ (\upsilon_2, \langle 0.82e^{2i\pi(0.81)}, 0.63e^{2i\pi(0.67)} \rangle) \\ (\upsilon_3, \langle 0.64e^{2i\pi(0.69)}, 0.81e^{2i\pi(0.42)} \rangle) \\ (\upsilon_4, \langle 0.58e^{2i\pi(0.65)}, 0.85e^{2i\pi(0.45)} \rangle) \end{Bmatrix}$$

$$F(\widetilde{M}_3) = \begin{Bmatrix} (\upsilon_1, \langle 0.60e^{2i\pi(0.40)}, 0.90e^{2i\pi(0.65)} \rangle) \\ (\upsilon_2, \langle 0.86e^{2i\pi(0.53)}, 0.55e^{2i\pi(0.65)} \rangle) \\ (\upsilon_3, \langle 0.45e^{2i\pi(0.65)}, 0.90e^{2i\pi(0.30)} \rangle) \\ (\upsilon_4, \langle 0.67e^{2i\pi(0.86)}, 0.80e^{2i\pi(0.65)} \rangle) \end{Bmatrix}$$

$$F(\widetilde{M}_4) = \begin{Bmatrix} (\upsilon_1, \langle 0.60e^{2i\pi(0.70)}, 0.85e^{2i\pi(0.75)} \rangle) \\ (\upsilon_2, \langle 0.87e^{2i\pi(0.70)}, 0.51e^{2i\pi(0.65)} \rangle) \\ (\upsilon_3, \langle 0.93e^{2i\pi(0.95)}, 0.47e^{2i\pi(0.16)} \rangle) \\ (\upsilon_4, \langle 0.79e^{2i\pi(0.85)}, 0.65e^{2i\pi(0.10)} \rangle) \end{Bmatrix}$$

2. Rationalized the CFFHSS for each alternatives is given in Table 3.

3. Ideal CFFHSS matrix value is shown in Table 4.

4. The Calculated Similarity measure $S_H(\upsilon,\upsilon_i)$, $S_E(\upsilon,\upsilon_i)$ and $S_V(\upsilon,\upsilon_i)$ given in definitions (6), (7) and (8) are shown in Table 5.

5. The Mean Value all Similarity measures are shown in Table 6.

6. Ranking of alternatives according to obtained similarity measures

| CFFHSS DV | $\boldsymbol{\upsilon_1}$ | $\boldsymbol{\upsilon_2}$ | $\boldsymbol{\upsilon_3}$ | $\boldsymbol{\upsilon_4}$ |
|---|---|---|---|---|
| $\boldsymbol{\widetilde{M}_1}$ | $\begin{pmatrix}0.70^{0.44}\\0.79^{0.67}\end{pmatrix}$ | $\begin{pmatrix}0.81^{0.75}\\0.66^{0.29}\end{pmatrix}$ | $\begin{pmatrix}0.47^{0.65}\\0.92^{0.30}\end{pmatrix}$ | $\begin{pmatrix}0.85^{0.46}\\0.68^{0.67}\end{pmatrix}$ |
| $\boldsymbol{\widetilde{M}_2}$ | $\begin{pmatrix}0.65^{0.69}\\0.81^{0.42}\end{pmatrix}$ | $\begin{pmatrix}0.82^{0.81}\\0.63^{0.67}\end{pmatrix}$ | $\begin{pmatrix}0.64^{0.69}\\0.81^{0.42}\end{pmatrix}$ | $\begin{pmatrix}0.58^{0.65}\\0.85^{0.45}\end{pmatrix}$ |
| $\boldsymbol{\widetilde{M}_3}$ | $\begin{pmatrix}0.60^{0.40}\\0.90^{0.65}\end{pmatrix}$ | $\begin{pmatrix}0.86^{0.53}\\0.55^{0.65}\end{pmatrix}$ | $\begin{pmatrix}0.45^{0.65}\\0.90^{0.30}\end{pmatrix}$ | $\begin{pmatrix}0.67^{0.86}\\0.80^{0.65}\end{pmatrix}$ |
| $\boldsymbol{\widetilde{M}_4}$ | $\begin{pmatrix}0.60^{0.70}\\0.80^{0.75}\end{pmatrix}$ | $\begin{pmatrix}0.87^{0.70}\\0.51^{0.65}\end{pmatrix}$ | $\begin{pmatrix}0.93^{0.95}\\0.47^{0.10}\end{pmatrix}$ | $\begin{pmatrix}0.79^{0.85}\\0.65^{0.10}\end{pmatrix}$ |

Table 3: Rationalize the CFFHSS Decision Values

| CFFHSS DV | $\upsilon$ |
|---|---|
| $\widetilde{M}_1$ | $\begin{pmatrix}0.85^{0.46}\\0.68^{0.67}\end{pmatrix}$ |
| $\widetilde{M}_2$ | $\begin{pmatrix}0.82^{0.81}\\0.63^{0.67}\end{pmatrix}$ |
| $\widetilde{M}_3$ | $\begin{pmatrix}0.86^{0.53}\\0.55^{0.65}\end{pmatrix}$ |
| $\widetilde{M}_4$ | $\begin{pmatrix}0.93^{0.95}\\0.47^{0.10}\end{pmatrix}$ |

Table 4: Ideal CFFHSS Decision Values

| **Similarity Measures** | $(\boldsymbol{v},\boldsymbol{v_1})$ | $(\boldsymbol{v},\boldsymbol{v_2})$ | $(\boldsymbol{v},\boldsymbol{v_3})$ | $(\boldsymbol{v},\boldsymbol{v_4})$ |
|---|---|---|---|---|
| $\boldsymbol{S_H(u,u_i)}$ | 0.5875 | 0.7962 | 0.6075 | 0.7462 |
| $\boldsymbol{S_E(u,u_i)}$ | 0.8646 | 0.9249 | 0.8820 | 0.9438 |
| $\boldsymbol{S_V(u,u_i)}$ | 0.9428 | 0.9673 | 0.9610 | 0.9866 |

Table 5: Ideal CFFHSS Decision Values

| $\boldsymbol{S(v,v_i)}$ | $\boldsymbol{v_1}$ | $\boldsymbol{v_2}$ | $\boldsymbol{v_3}$ | $\boldsymbol{v_4}$ |
|---|---|---|---|---|
| $\boldsymbol{v}$ | 0.7983 | 0.8961 | 0.8168 | 0.8922 |

Table 6: Mean of Similarity Measures

7. As the alternative $v_2$ has the highest similarity measure with the ideal alternative, therefore, the best alternative from available data will be $u_2$.

## 3.2 Sensitivity Analysis

1. Geometric Mean

| **Geometric Mean** | $(\boldsymbol{v},\boldsymbol{v_1})$ | $(\boldsymbol{v},\boldsymbol{v_2})$ | $(\boldsymbol{v},\boldsymbol{v_3})$ | $(\boldsymbol{v},\boldsymbol{v_4})$ |
|---|---|---|---|---|
| $\boldsymbol{v}$ | 0.7820 | 0.8920 | 0.8010 | 0.8840 |

Table 7: Geometric Mean of CFFHSS Decision Values

2. Harmonic Mean

| **Harmonic Mean** | $(\boldsymbol{v}, \boldsymbol{v_1})$ | $(\boldsymbol{v}, \boldsymbol{v_2})$ | $(\boldsymbol{v}, \boldsymbol{v_3})$ | $(\boldsymbol{v}, \boldsymbol{v_4})$ |
|---|---|---|---|---|
| $\boldsymbol{v}$ | 0.7657 | 0.8900 | 0.7859 | 0.8787 |

Table 8: Harmonic Mean of CFFHSS Decision Values

By Evaluating the Harmonic mean , Geometric mean it is concluded that $v_2$ is best availiable alternative. So Our purposed algorithm is accurate.

# 4 Conclusion

In this paper Complex farmateen fuzzy hyper soft set(CFFHSS) is introduced which not only dealt with parameters but also with sub-parametrs. Some Similarity formulas are defined for CFFHSS, Which helps us to measure similarity between two CFFHSS. Further for the selection of best available mobile phone a thorough algorithm is given on the basis of these similarity measures and by this purposed algorithms we can select best available Mobile phone. For this a case study is also discussed in this paper, on which given algorithm is applied, and to show that our purposed algorithm is effective and fruitful A statistical analysis is also applied, which will tell us that how our purposed algorithm is accurate. after these discussions the paper is finally concluded with complete synthesis and future direction with limitations of our structure. The limitation of our structures are that CFFHSS is defined only for those Numbers where $0 < r_{\Theta F}^3 + r_{\Xi F}^3 < 1$ and $0 < \vartheta_{\Theta F}^3 + \vartheta_{\Xi F}^3 < 1$. and also with condition $\vartheta_{\Theta F}^2 + \vartheta_{\Xi F}^2 \geq 1$ . Future Researchers can work on these limitations to obtain results more quicker and accurate.

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