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A Multi-Attribute Decision-Making Technique for Supplier Selection Using Complex Picture Fuzzy Hypersoft Set

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Abstract: *This research deals with the complexities of multi-attribute decision making (MADM) under uncertainty, where traditional models struggle to handle imprecise and contravening data. To overcome these restrictions a novel framework, Complex Picture Fuzzy Hypersoft Set (CPicFHySS) is acquainted, which incorporates complex fuzzy domain along with picture fuzzy and hypersoft sets to capture multidimensional uncertainty. This research starts with the new hybrid structure CPicFHySS, along with its some set-theoretic properties. Further using newly constructed structure an MADM-based algorithm is presented, which improves the results of multi-criteria decision making process. The real-world case-study validates the newly presented structure, demonstrating its value in handling uncertainty and ambiguity in daily life. Finally, to highlight the superiority of CPicFHySS a comparative* *analysis with already existing models is presented. Under uncertain environments this research gives a solid mathematical foundation for decision-making, with potential applications in engineering, business analytics, supplier and energy portfolios selection.*

Keywords: *fuzzy soft set; imprecision and uncertainty; multi-attribute decision making (MADM); complex picture fuzzy hypersoft set; supplier chain selection*

1. Introduction

Decision making process face complexities, to tackle them a modern multi-attribute decision-making (MADM) method have been deployed, which gives more holistic framework for analyzing interconnected variables, that allows for a comprehensive analysis of competing factors.[1-4] However, these methods still struggles with the ambiguous, incomplete and dynamically changing data. Examples includes medical diagnosis with inconclusive symptoms and test results. Another example is financial markets where trends fluctuate unpredictably. These challenges spurred the evolution of MADM techniques[5, 6].

Mathematical frameworks have improved significantly over the years to deal with uncertain situations more effectively. The foundation was laid by Zadeh's (1965)[7] fuzzy set theory, which introduced the partial membership concept, which allowed values between 0 and 1 to represent truth degree. Atanassov (1983)[8] expanded this idea with Intuitionistic fuzzy sets (InFS) by adding a non-membership degree to catch the opposing indications, which improved ambiguity representation in decision-making. Afterwards, Cuong et al. (2013)[9] developed picture fuzzy sets (PicFS), which includes a neutral membership attribute to account for decision-making hesitation.

Meanwhile, a different approach was introduced which handles the periodic uncertainty, when Ramot et al. (2002)[10] gave the concept of complex fuzzy sets (CFS), in which membership values are represented as complex numbers which enables phase-based modeling of periodic data. This was further processed into complex intuitionistic fuzzy sets (CInFS) by Alkouri & Salleh (2012)[11] and complex picture fuzzy sets (CPicFS) by Akram et al. (2020)[12] through the addition of complex domain to intuitionistic fuzzy sets and picture fuzzy sets respectively. It enhance the ability to analyze systems with repeating patterns, such as seasonal trends in economics or wave-based phenomena in engineering field.

Parallel to these improvements, soft set theory (Molodtsov, 1999)[13] developed a parameterized approach to decision-making, allowing flexible analysis of attributes. This led to some more nuanced extensions like fuzzy soft sets (Maji, 2001-a)[14], intuitionistic fuzzy soft sets (Maji et al., 2010-b)[15], and picture fuzzy soft sets (Yang, 2015)[16], each structure improved the uncertainty handling with addition of soft set concept in proceeding ones. Thirunavukarasu et al. (2017)[17] modified the concept of soft set theory by introducing new structure named as Complex Fuzzy Soft Sets (CFSS), in which complex-valued membership functions jointly record amplitude and phase term. Complex Intuitionistic Fuzzy Soft Set (CIFSS) was introduced by the Kumar and Bajaj [18] in the year 2014 which combines the concepts of complex, intuitionistic and soft set to manage uncertainty, dual membership and periodicity. In a new development in this field , Tahir Mahmood and Ubaid ur Rehman (2021) [19] presented an improved definition of Complex Picture Fuzzy Soft Set (CPicFSS) by combination of complex fuzzy sets with picture fuzzy sets. A major breakthrough came with Smarandacheâ (2018) [20] in form of hypersoft sets, which modified multi-dimensional attribute classification, making it possible to dissect complex problems into finer, more accomplishable components. Recent innovations, such as complex fuzzy hypersoft sets (CFHySS) (Rehman et al., 2020)[21], combined complex-valued memberships with multi-parameter analysis, offering a more robust framework for dynamic decision-making.

Despite these advancements, existing models still struggle with high-dimensional uncertainty in multi-attribute decision-making (MADM) problems. Many real-world scenarios such as supplier chain selection the reliability factor often fluctuate, similarly in selection of energy portfolio output metrics changes with time to time. In healthcare diagnostics with incomplete patient records require a more adaptable and precise approach[22-24]. To bridge this gap, this paper introduces a novel hybrid structure called Complex Picture Fuzzy Hypersoft Set (CPicFHySS), which integrates three powerful concepts, firstly the complex-valued memberships by capturing periodic and phase-based uncertainty, secondly Picture fuzzy logic which models positive, negative and neutral preferences and thirdly hypersoft attributes that enables multi-criteria analysis.

1.1 Arrangement of Article

This article introduces a new structure i.e Complex Picture Fuzzy Hypersoft Set (CPicFHySS), an advanced mathematical framework that synthesizes the descriptive capacity of picture fuzzy sets with the multi-argument functionality of hypersoft sets. The steps of the work are as given, Section I which is Introduction; includes the literature study, and point outs the targets that existing models lacks in specific research. Section II named as fundamentals of CPicFHySS; furnishes the essential foundational concepts of new model by combining the definitions associated with fuzzy, complex and hypersoft set-like structures. Section III is dedicated to the core concept of the CPicFHySS model, it demonstrates the model's utility in applied settings by deploying its algorithms; which includes an MADM methodology applied in case study concerning supplier selection. Section IV which is comparision, delivers a critical evaluation of the CPicFHySS, presenting a systematic structural comparison against a series of existing models, validating its effectiveness and robustness. Finally, the concluding section VI, consolidates the essential contributions, concedes limitations and outlines a research roadmap, including dynamic parameter optimization and hybridized computational techniques.

2. Fundamentals of Complex Picture Fuzzy Hypersoft Set (CPicFHySS)

2.1 Preliminaries

To understanding the main concept of this article let's recall some of the basic definitions from literature.

Definition 1. *Fuzzy Set* *[7]: Let U be a universal set. A fuzzy set (FS) over U is defined by a membership function:*

$$\alpha: U \rightarrow [0,1],$$

where $\alpha(u)$ represents the degree of membership of $u \in U$. The set is represented by,

$$FS = \{\langle u, \alpha(u)\rangle | u \in U\}.$$

Definition 2. *Intuitionistic Fuzzy Set* *[8]: An intuitionistic fuzzy set (InFS) over U is defined by two functions: $\alpha: U \rightarrow [0,1]$ (membership), $\beta: U \rightarrow [0,1]$ (non-membership), such that for all $u \in U$,*

$$0 \leq \alpha(u) + \beta(u) \leq 1.$$

The set is represented by,

$$InFS = \{\langle u, \alpha(u), \beta(u)\rangle | u \in U\}$$

Definition 3. ***Picture Fuzzy Set*** *[9]: A picture fuzzy set (PicFS) over U extends InFS by adding a neutral membership function $\gamma: U \to [0,1]$. It is defined by,*

$$PicFS = \{\langle u, \alpha(u), \beta(u), \gamma(u)\rangle | u \in U\},$$

with the constraints,

$$0 \leq \alpha(u) + \beta(u) + \gamma(u) \leq 1.$$

Definition 4. ***Soft Set*** *[13]: Let U be a universal set and E a set of attributes. A soft set (SS) over U is a pair (Δ, E), where,*

$$\Delta: E \to \mathcal{P}(U),$$

which assigns a subset of U to each attribute $e \in E$. It is written by,

$$SS = \{\langle e, \Delta(e)\rangle | e \in E\}$$

Definition 5. ***Hypersoft Set*** *[20]: Let $\{A_k\}_{k=1}^{m}$ be disjoint attribute sets, and $\Omega = A_1 \times \cdots \times A_m$. A hypersoft set (HySS) over U is a pair (Δ, Ω), where,*

$$\Delta: \Omega \to \mathcal{P}(U),$$

maps each parameter combination $c = (a_{1p}, \dots, a_{mr}) \in \Omega$ to a subset of U.

Definition 6. ***Complex Fuzzy Hypersoft Set*** *[21]: A complex fuzzy hypersoft set (CFHySS) over U is a pair (Δ, Ω), such that,*

$$\Delta: \Omega \to \mathbb{C}(U),$$

which assigns a complex fuzzy set ψ_c to each $c \in \Omega$. Here, $\mathbb{C}(U)$ denotes all complex fuzzy sets on U, and

$$\psi_c(u) = \rho_c(u)\exp^{i\theta_c(u)}, \quad \rho_c(u) \in [0,1], \quad \theta_c(u) \in [0,2\pi]$$

2.2 Complex Picture Fuzzy Hypersoft Set ($CPicFHySS$)

In this section the definition of new framework CPicFHySS along with example is presented.

Definition 7. *Let U be a universal set, and $A_1, A_2, \dots, A_m$ be distinct attribute sets such that $A_i \cap A_j = \emptyset$ for $i \neq j$. The Cartesian product of these attribute sets is denoted by $\Omega = A_1 \times A_2 \times \cdots \times A_m$. A mapping $\Delta: \Omega \to CPF(U)$ assigns to each parameter $c = (a_{1p}, a_{2q}, \dots, a_{mr}) \in \Omega$ a Complex Picture Fuzzy (CPF) subset of U. Here, $CPF(U)$ represents the collection of all CPF subsets over U. The pair (Δ, Ω) is called a Complex Picture Fuzzy Hypersoft Set ($CP_{ic}FH_ySS$) on U, where for any $c \in \Omega$, $\Delta(c)$ is defined as,*

$$\Delta(c) = \{\langle u, \alpha, \beta, \gamma \rangle \in U \times \mathcal{C}_1 \times \mathcal{C}_1 \times \mathcal{C}_1 | \alpha = \alpha_c(u), \beta = \beta_c(u), \gamma = \gamma_c(u) \ \wedge \ \mathcal{C}_1 = |z| \leq 1\}$$

$\alpha_c(u) = \rho^+(c)exp^{i\theta(c)}$ is the complex membership,$\beta_c(u) = \rho^-(c)exp^{i\phi(c)}$ is complex non-membership and $\gamma_c(u) = \rho(c)exp^{i\psi(c)}$ is complex neutral grade with the following conditions,

$$0 \leq \rho^+(c) + \rho^-(c) + \rho(c) \leq 1, 0 \leq \theta(c) + \phi(c) + \psi(c) \leq 2\pi$$

for the convenience we write $(\alpha e^{i\theta\pi}, \beta e^{i\phi\pi}, \gamma e^{i\psi\pi})$ as $(\alpha, \beta, \gamma)e^{i(\theta,\phi,\psi)\pi}$. It can be represented in block matrix as,

$$\hat{S}_{h\times k} = \begin{bmatrix} \begin{vmatrix} \alpha_{11}e^{\theta_{11}} \\ \beta_{11}e^{\phi_{11}} \\ \gamma_{11}e^{\psi_{11}} \end{vmatrix} & \begin{vmatrix} \alpha_{12}e^{\theta_{12}} \\ \beta_{12}e^{\phi_{12}} \\ \gamma_{12}e^{\psi_{12}} \end{vmatrix} & \cdots & \begin{vmatrix} \alpha_{1k}e^{\theta_{1k}} \\ \beta_{1k}e^{\phi_{1k}} \\ \gamma_{1k}e^{\psi_{1k}} \end{vmatrix} \\ \begin{vmatrix} \alpha_{21}e^{\theta_{21}} \\ \beta_{21}e^{\phi_{21}} \\ \gamma_{21}e^{\psi_{21}} \end{vmatrix} & \begin{vmatrix} \alpha_{22}e^{\theta_{22}} \\ \beta_{22}e^{\phi_{22}} \\ \gamma_{22}e^{\psi_{22}} \end{vmatrix} & \cdots & \begin{vmatrix} \alpha_{2k}e^{\theta_{2k}} \\ \beta_{2k}e^{\phi_{2k}} \\ \gamma_{2k}e^{\psi_{2k}} \end{vmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \begin{vmatrix} \alpha_{h1}e^{\theta_{h1}} \\ \beta_{h1}e^{\phi_{h1}} \\ \gamma_{h1}e^{\psi_{h1}} \end{vmatrix} & \begin{vmatrix} \alpha_{h2}e^{\theta_{h2}} \\ \beta_{h2}e^{\phi_{h2}} \\ \gamma_{h2}e^{\psi_{h2}} \end{vmatrix} & \cdots & \begin{vmatrix} \alpha_{hk}e^{\theta_{hk}} \\ \beta_{hk}e^{\phi_{hk}} \\ \gamma_{hk}e^{\psi_{hk}} \end{vmatrix} \end{bmatrix}$$

Example 8. *Let $U = \{\mathbb{S}_1, \mathbb{S}_2, \mathbb{S}_3\}$ be a set of smartphones, and consider the following attribute sets: $\mathcal{A}_1 = Price = \{Low, Medium, High\}$, $\mathcal{A}_2 = Camera = \{Basic, Advanced\}$. $\mathcal{P} = \mathcal{A}_1 \times \mathcal{A}_2$ has six combinations. The Cartesian product $\Omega = A_1 \times A_2$ yields six combinations,*

$$\Omega = \left\{ \begin{array}{c} c_1 = (\text{Low, Basic}), c_2 = (\text{Low, Advance}), c_3 = (\text{Medium, Basic}), \\ c_4 = (\text{Medium, Advance}), c_5 = (\text{High, Basic}), c_6 = (\text{High, Advance}) \end{array} \right\}$$

$$(\Delta, \Omega) = \{\Delta(c_1), \Delta(c_2), \Delta(c_3) \dots, \Delta(c_6)\}$$

For $c_1 = (\text{Low, Basic})$:

$$\Delta(c_1) = \left\{ \frac{\langle 0.6,0.1,0.2\rangle exp^{i\langle 0.5,0.2,0.3\rangle\pi}}{\mathbb{S}_1}, \frac{\langle 0.6,0.3,0.1\rangle exp^{i\langle 0.5,0.2,0.3\rangle\pi}}{\mathbb{S}_2}, \frac{\langle 0.6,0.2,0.1\rangle exp^{i\langle 0.7,0.6,0.2\rangle\pi}}{\mathbb{S}_3} \right\}$$

For $c_2 = (\text{Low, Advance})$:

$$\Delta(c_2) = \left\{ \frac{\langle 0.5,0.2,0.2\rangle exp^{i\langle 0.3,0.4,0.4\rangle\pi}}{\mathbb{S}_1}, \frac{\langle 0.7,0.2,0.1\rangle exp^{i\langle 0.6,0.3,0.6\rangle\pi}}{\mathbb{S}_2}, \frac{\langle 0.4,0.2,0.3\rangle exp^{i\langle 0.8,0.4,0.2\rangle\pi}}{\mathbb{S}_3} \right\}$$

For $c_3 = (\text{Medium, Basic})$:

$$\Delta(c_3) = \left\{ \frac{\langle 0.7,0.1,0.1\rangle exp^{i\langle 0.6,0.4,0.5\rangle\pi}}{\mathbb{S}_1}, \frac{\langle 0.6,0.2,0.1\rangle exp^{i\langle 0.5,0.4,0.6\rangle\pi}}{\mathbb{S}_2}, \frac{\langle 0.5,0.2,0.1\rangle exp^{i\langle 0.9,0.2,0.2\rangle\pi}}{\mathbb{S}_3} \right\}$$

Above data can be represented in matrix form as,

$$\mathcal{H} = \begin{bmatrix} \left\langle \begin{matrix} 0.60e^{0.50} \\ 0.10e^{0.20} \\ 0.20e^{0.30} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.60e^{0.50} \\ 0.30e^{0.20} \\ 0.10e^{0.30} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.60e^{0.70} \\ 0.20e^{0.60} \\ 0.10e^{0.20} \end{matrix} \right\rangle \\ \left\langle \begin{matrix} 0.50e^{0.30} \\ 0.20e^{0.40} \\ 0.20e^{0.40} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.70e^{0.60} \\ 0.20e^{0.30} \\ 0.10e^{0.60} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.40e^{0.80} \\ 0.20e^{0.40} \\ 0.30e^{0.20} \end{matrix} \right\rangle \\ \left\langle \begin{matrix} 0.70e^{0.60} \\ 0.10e^{0.40} \\ 0.10e^{0.50} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.60e^{0.50} \\ 0.20e^{0.40} \\ 0.10e^{0.60} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.50e^{0.90} \\ 0.20e^{0.20} \\ 0.10e^{0.20} \end{matrix} \right\rangle \end{bmatrix}$$

2.3 Some Set-Theoretical Properties of CPicFHySS

• CPicFHySS Union:

Let (Δ_1, Ω_1) and (Δ_2, Ω_2) be two CPiFHSS over U. Their union $(\Delta, \Omega) = (\Delta_1, \Omega_1) \cup (\Delta_2, \Omega_2)$ For each $c \in \Omega$ is defined by,

$$\Delta(c) = \begin{cases} \Delta_1(c) & \text{if} c \in \Omega_1 - \Omega_2, \\ \Delta_2(c) & \text{if} c \in \Omega_2 - \Omega_1, \\ \Delta_1(c) \sqcup \Delta_2(c) & \text{if} c \in \Omega_1 \sqcap \Omega_2, \end{cases}$$

can be expressed as,

$$(\alpha, \beta, \gamma)e^{i(\theta,\phi,\psi)\pi} \cup (\alpha', \beta', \gamma')e^{i(\theta\prime,\phi\prime,\psi\prime)\pi}$$
$$= (\max(\alpha, \alpha'), \min(\beta, \beta'), \min(\gamma, \gamma'))e^{i(\max(\theta,\theta\prime),\min(\phi,\phi\prime),\min(\psi,\psi\prime))\pi}$$

• CPicFySS Intersection:

Let (Δ_1, Ω_1) and (Δ_2, Ω_2) be two CPiFHSS over U, then their intersection

$(\Delta, \Omega) = (\Delta_1, \Omega_1) \cap (\Delta_2, \Omega_2)$ is defined for for each $c \in \Omega$ by, $\Delta(c) = \Delta_1(c) \cap \Delta_2(c)$

$$(\alpha, \beta, \gamma)e^{i(\theta,\phi,\psi)\pi} \cap (\alpha', \beta', \gamma')e^{i(\theta\prime,\phi\prime,\psi\prime)\pi}$$
$$= (\min(\alpha, \alpha'), \max(\beta, \beta'), \max(\gamma, \gamma'))e^{i(\min(\theta,\theta\prime),\max(\phi,\phi\prime),\max(\psi,\psi\prime))\pi}$$

Example 9

$$\Delta_1(c) \cup \Delta_2(c)$$
$$= \left\{ \frac{\langle 0.6,0.1,0.2\rangle exp^{i\langle 0.5,0.2,0.3\rangle\pi}}{S_1}, \frac{\langle 0.7,0.2,0.1\rangle exp^{i\langle 0.6,0.2,0.3\rangle\pi}}{S_2}, \frac{\langle 0.6,0.2,0.1\rangle exp^{i\langle 0.8,0.4,0.2\rangle\pi}}{S_3} \right\}$$

Example 10

$$\Delta_1(c) \cap \Delta_2(c)$$
$$= \left\{ \frac{\langle 0.5,0.2,0.2\rangle exp^{i\langle 0.3,0.4,0.4\rangle\pi}}{S_1}, \frac{\langle 0.6,0.3,0.1\rangle exp^{i\langle 0.5,0.3,0.6\rangle\pi}}{S_2}, \frac{\langle 0.4,0.2,0.3\rangle exp^{i\langle 0.7,0.6,0.2\rangle\pi}}{S_3} \right\}$$

3. MADM-based Algorithm for CPicFHySS

Step 1: Defining Problem and Data Collection

Identify the universal set U of alternatives and the disjoint attribute sets $A_1, A_2, \ldots, A_n$, and construct the parameter set $\Omega = A_1 \times A_2 \times \cdots \times A_n$. Collect expert evaluations for each alternative under each parameter in terms of complex picture fuzzy numbers (CPiFNs).

Step 2: Construct the CPicFHySS

For each parameter $c \in \Omega$, define the mapping $\Delta(c)$ as a CPiF subset over U. Represent the CPicFHySS (Δ, Ω) in matrix form for computational ease.

$$\hat{S}_{h\times k} = \begin{bmatrix} \left\langle\begin{matrix}\alpha_{11}e^{\theta_{11}}\\ \beta_{11}e^{\phi_{11}}\\ \gamma_{11}e^{\psi_{11}}\end{matrix}\right\rangle & \left\langle\begin{matrix}\alpha_{12}e^{\theta_{12}}\\ \beta_{12}e^{\phi_{12}}\\ \gamma_{12}e^{\psi_{12}}\end{matrix}\right\rangle & \cdots & \left\langle\begin{matrix}\alpha_{1k}e^{\theta_{1k}}\\ \beta_{1k}e^{\phi_{1k}}\\ \gamma_{1k}e^{\psi_{1k}}\end{matrix}\right\rangle \\ \left\langle\begin{matrix}\alpha_{21}e^{\theta_{21}}\\ \beta_{21}e^{\phi_{21}}\\ \gamma_{21}e^{\psi_{21}}\end{matrix}\right\rangle & \left\langle\begin{matrix}\alpha_{22}e^{\theta_{22}}\\ \beta_{22}e^{\phi_{22}}\\ \gamma_{22}e^{\psi_{22}}\end{matrix}\right\rangle & \cdots & \left\langle\begin{matrix}\alpha_{2k}e^{\theta_{2k}}\\ \beta_{2k}e^{\phi_{2k}}\\ \gamma_{2k}e^{\psi_{2k}}\end{matrix}\right\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \left\langle\begin{matrix}\alpha_{h1}e^{\theta_{h1}}\\ \beta_{h1}e^{\phi_{h1}}\\ \gamma_{h1}e^{\psi_{h1}}\end{matrix}\right\rangle & \left\langle\begin{matrix}\alpha_{h2}e^{\theta_{h2}}\\ \beta_{h2}e^{\phi_{h2}}\\ \gamma_{h2}e^{\psi_{h2}}\end{matrix}\right\rangle & \cdots & \left\langle\begin{matrix}\alpha_{hk}e^{\theta_{hk}}\\ \beta_{hk}e^{\phi_{hk}}\\ \gamma_{hk}e^{\psi_{hk}}\end{matrix}\right\rangle \end{bmatrix}$$

Step 3: Compute the Factor Matrix

Transform the CPicFHySS matrix into a factor matrix by aggregating the membership, non-membership, and neutral grades for each alternative. For each alternative $u \in U$ and parameter $c \in \Omega$, now compute,

$$\text{FactorValue} = \delta_c(u) = |\rho^+(c) + \rho(c) - \rho^-(c)| \cdot e^{i|\theta(u)+\psi(u)-\phi(u)|}$$

$$\widehat{\Omega}^{\text{factor}}_{h\times k} = \begin{bmatrix} \delta_c(u)_{11} & \delta_c(u)_{12} & \cdots & \delta_c(u)_{1k} \\ \delta_c(u)_{21} & \delta_c(u)_{22} & \cdots & \delta_c(u)_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_c(u)_{h1} & \delta_c(u)_{h2} & \cdots & \delta_c(u)_{hk} \end{bmatrix}$$

Step 4: Split the factor matrix into grades and period components.

$$\widehat{\Omega}_{grade}^{\text{factor}} = \begin{bmatrix} r_c(u)_{11} & r_c(u)_{12} & \cdots & r_c(u)_{1k} \\ r_c(u)_{21} & r_c(u)_{22} & \cdots & r_c(u)_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ r_c(u)_{h1} & r_c(u)_{h2} & \cdots & r_c(u)_{hk} \end{bmatrix}$$

$$\widehat{\Omega}_{grade}^{\text{factor}} = \begin{bmatrix} \Theta_c(u)_{11} & \Theta_c(u)_{12} & \cdots & \Theta_c(u)_{1k} \\ \Theta_c(u)_{21} & \Theta_c(u)_{22} & \cdots & \Theta_c(u)_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \Theta_c(u)_{h1} & \Theta_c(u)_{h2} & \cdots & \Theta_c(u)_{hk} \end{bmatrix}$$

Step 5: Calculate Judgement Values

For each alternative, compute the highest and lowest judgemental values for both grades and periods terms.

$$\mathbb{J}_{\text{grade}}^{\text{high}}(u) = \sum_{c\in\Omega} (1 - r_c(u))^2, \quad \mathbb{J}_{\text{grade}}^{\text{low}}(u) = \sum_{c\in\Omega} (r_c(u))^2$$

$$\mathbb{J}_{\text{period}}^{\text{high}}(u) = \sum_{c\in\Omega} (1 - \Theta_c(u))^2, \quad \mathbb{J}_{\text{period}}^{\text{low}}(u) = \sum_{c\in\Omega} (\Theta_c(u))^2$$

Compute the rating score.

$$\Re_{\text{grade}}(u) = \frac{\mathbb{J}_{\text{grade}}^{\text{high}}(u) + \mathbb{J}_{\text{grade}}^{\text{low}}(u)}{|\Omega|}, \quad \Re_{\text{period}}(u) = \frac{\mathbb{J}_{\text{period}}^{\text{high}}(u) + \mathbb{J}_{\text{period}}^{\text{low}}(u)}{|\Omega|}$$

Step 6: Rank the Alternatives

Compute the mean rating score for each alternative.

$$\Re(u) = \frac{\Re_{\text{grade}}(u) + \Re_{\text{period}}(u)}{2}$$

Rank the alternatives in descending order of $\mathfrak{R}(u)$. The alternative with the highest rating score is the optimal choice.

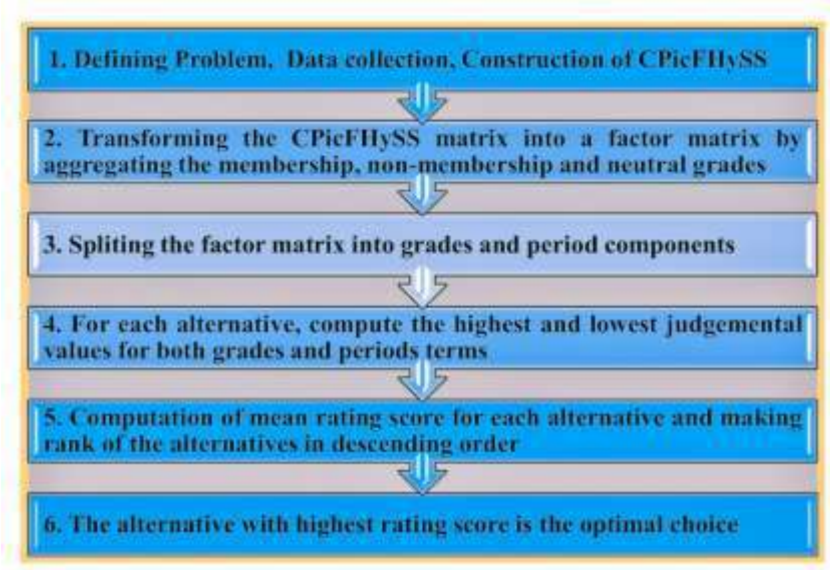

Figure 1: Step wise algorithmic approach

3.1 Case-Study to Select the Most Suitable Supplier

A manufacturing company aims to select the most suitable supplier for raw materials to ensure quality, cost-effectiveness and reliability in its supply chain. The company evaluates four potential suppliers which are given as local supplier ($\mathfrak{S}_1$), international supplier ($\mathfrak{S}_2$), eco-friendly supplier ($\mathfrak{S}_3$), bulk discount supplier ($\mathfrak{S}_4$), i.e $U = \{\mathfrak{S}_1, \mathfrak{S}_2, \mathfrak{S}_3, \mathfrak{S}_4\}$. The selection is based on three key criteria, $\hat{x}_1$=(Cost Efficiency) with sub-attributes of Low, Medium & High. $\hat{x}_2$=(Quality Assurance) with sub-attributes of Excellent, Average & Poor. $\hat{x}_3$=(Delivery Reliability) with sub-attributes of On-Time, Delayed. The parameter set is given by, $\Omega = \hat{x}_1 \times \hat{x}_2 \times \hat{x}_3$ yielding 27 possible combinations. A committee of four experts evaluates the suppliers which is given under. $\tilde{y}_1$=Procurement Manager (focuses on cost), $\tilde{y}_2$=Quality Inspector (focuses on material quality), $\tilde{y}_3$=Logistics Head (focuses on delivery performance), $\tilde{y}_4$=Sustainability Officer (focuses on eco-friendly practices). For illustration, consider the subset $\epsilon = \{c_1, c_5, c_9, c_{12}\} \subseteq \Omega$, the data is represented in matrix form as,

$$\widehat{\Omega}_{4\times4}^{\text{dataset}} = \begin{bmatrix} \left\langle \begin{matrix} 0.71e^{0.96} \\ 0.18e^{0.44} \\ 0.11e^{0.37} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.63e^{0.76} \\ 0.13e^{0.57} \\ 0.24e^{0.44} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.67e^{0.76} \\ 0.18e^{0.44} \\ 0.11e^{0.27} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.48e^{0.44} \\ 0.39e^{0.57} \\ 0.12e^{0.36} \end{matrix} \right\rangle \\ \left\langle \begin{matrix} 0.48e^{0.56} \\ 0.37e^{0.47} \\ 0.07e^{0.18} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.47e^{0.67} \\ 0.31e^{0.26} \\ 0.21e^{0.18} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.58e^{0.87} \\ 0.27e^{0.37} \\ 0.13e^{0.24} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.63e^{0.96} \\ 0.17e^{0.28} \\ 0.18e^{0.24} \end{matrix} \right\rangle \\ \left\langle \begin{matrix} 0.44e^{0.56} \\ 0.42e^{0.67} \\ 0.13e^{0.78} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.82e^{0.94} \\ 0.09e^{0.66} \\ 0.08e^{0.37} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.75e^{0.76} \\ 0.14e^{0.44} \\ 0.11e^{0.56} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.72e^{0.76} \\ 0.11e^{0.48} \\ 0.14e^{0.52} \end{matrix} \right\rangle \\ \left\langle \begin{matrix} 0.22e^{0.54} \\ 0.57e^{0.77} \\ 0.18e^{0.33} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.55e^{0.87} \\ 0.25e^{0.58} \\ 0.15e^{0.36} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.24e^{0.76} \\ 0.48e^{0.67} \\ 0.27e^{0.48} \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.35e^{0.36} \\ 0.25e^{0.57} \\ 0.15e^{0.28} \end{matrix} \right\rangle \end{bmatrix}$$

Core Matrix Calculation For each supplier and parameter, compute the factor value,

$$\text{FactorValue} = |\alpha_c(u) + \gamma_c(u) - \beta_c(u)| \cdot e^{i|\theta(u)+\psi(u)-\phi(u)|}$$

$$\text{FactorValue} = |0.71 + 0.11 - 0.18| e^{i|0.96\pi+0.37\pi-0.44\pi|} = 0.64exp^{i0.89\pi}$$

$$\widehat{\Omega}_{4\times4}^{\text{factor}} = \begin{bmatrix} 0.64exp^{i0.89\pi} & 0.74exp^{i0.63\pi} & 0.60exp^{i0.59\pi} & 0.21exp^{i0.23\pi} \\ 0.18exp^{i0.27\pi} & 0.37exp^{i0.59\pi} & 0.44exp^{i0.74\pi} & 0.64exp^{i0.92\pi} \\ 0.15exp^{i0.67\pi} & 0.81exp^{i0.65\pi} & 0.72exp^{i0.88\pi} & 0.75exp^{i0.80\pi} \\ 0.17exp^{i0.10\pi} & 0.45exp^{i0.65\pi} & 0.03exp^{i0.57\pi} & 0.25exp^{i0.70\pi} \end{bmatrix}$$

Now split the core matrix into grade matrix and phase matrix separately,

$$\widehat{\Omega}_{grade}^{\text{factor}} = \begin{bmatrix} 0.64 & 0.74 & 0.60 & 0.21 \\ 0.18 & 0.37 & 0.44 & 0.64 \\ 0.15 & 0.81 & 0.72 & 0.75 \\ 0.17 & 0.45 & 0.03 & 0.25 \end{bmatrix}$$

$$\widehat{\Omega}_{period}^{\text{factor}} = \begin{bmatrix} 0.89 & 0.63 & 0.59 & 0.23 \\ 0.27 & 0.59 & 0.74 & 0.92 \\ 0.67 & 0.65 & 0.88 & 0.80 \\ 0.10 & 0.65 & 0.57 & 0.70 \end{bmatrix}$$

$\widehat{\Omega}_{4\times 4}^{\text{factor}}$	$\mathfrak{S}_1$	$\mathfrak{S}_2$	$\mathfrak{S}_3$	$\mathfrak{S}_4$
$\mathbb{J}_{\text{grade}}^{\text{high}}(\tilde{S}_m)$	0.9813	1.5115	0.8995	2.4948
$\mathbb{J}_{\text{grade}}^{\text{low}}(\tilde{S}_m)$	1.3613	0.7725	1.7595	0.2948
$\mathbb{J}_{\text{period}}^{\text{high}}(\tilde{S}_m)$	1.5900	0.6175	2.2858	0.4931
$\mathbb{J}_{\text{period}}^{\text{low}}(\tilde{S}_m)$	0.9100	1.8150	0.2858	1.2474
$\mathfrak{R}_{\text{grade}}(\tilde{S}_m)$	0.5856	0.5710	0.6647	0.6974
$\mathfrak{R}_{\text{period}}(\tilde{S}_m)$	0.3739	0.5965	0.9505	0.4862
$\mathfrak{R}_{\text{rating}}(\tilde{S}_m)$	0.4797	0.5837	**0.8076**	0.5918

Table 1: Judgement Values

The rating is $\mathfrak{S}_3 > \mathfrak{S}_4 > \mathfrak{S}_2 > \mathfrak{S}_1$, so $\mathfrak{S}_3$ is the best choice.

4. Comparison

Within the vast field of fuzzy set theory, unlike previous used models i.e InFS [8], CInFSS [18], PicFS [9], PicFSS [16], CPicFSS [19] and CFHySS [21], the presented model CPicFHySS gives a twisted and highly applicable mathematical framework. CPicFHySS introduces a novel integration of complex numbers with multi-attribute decision-making capabilities, making it more powerful tool in handling real-world uncertainty. It uniquely combines multiple boosted features, setting it apart from other related previous models. A defining characteristic of CPicFHySS is its use of complex-valued membership functions, which capture three distinct aspects of uncertainty i.e membership, non-membership and neutral degree, which are mathematically expressed by, $\alpha_c(u) = \rho^+(c)exp^{i\theta(c)}$, $\beta_c(u) = \rho^-(c)exp^{i\phi(c)}$, $\gamma_c(u) = \rho(c)exp^{i\psi(c)}$

By incorporating both magnitude and phase components, CPicFHySS provides a more significant representation of uncertainty, making it specially effective in analyzing real-world cyclical data patterns. Additionally, CPicFHySS employs a multi-attribute parametrization function given as, $f: A_1 \times A_2 \times \ldots \times A_n \rightarrow U$ This function allows CPicFHySS to process decision-making scenarios involving multiple interdependent attributes simultaneously, significantly enhancing its applicability in real-world problems.

In Contrast, the PicFHySS model boasts the similar multi-attribute handling abilities just like CPicFHySS but operates exclusively with real-valued membership degrees, but the absence of complex numbers simplifies its structure, making it less expressive in scenarios requiring phase-based analysis.

Conversely, though the CFHySS model utilizes complex-valued membership functions yet only considers a single membership attribute, ignoring the neutral and negative components. This restriction reduces its descriptive power as compared to CPiFHSS, especially in situations where multidimensional uncertainty assessment is required.

Considering the less complex approaches, the PicFSS and CPicFSS cater to single-attribute decision-making. PicFSS employs real-valued membership degrees, whereas CPicFSS enhances this with complex-valued functions. However, neither model supports multi-attribute parameterization, restricting their use to problems where decisions are based on a single criterion. Based on this comparison, CPicFHySS stands out as the most advanced and adoptable approach, providing superior accuracy in handling complex, multi-attribute decision-making challenges. Its aptitude to unify complex-valued uncertainty representation with multi-dimensional parametrization renders it a vital resource for experts in dealing with real-world and data-intensive problems.

Table 2: Structural Comparison

References	Mb	N.Mb	Neu	Mag	Phase	P.T	S.P.T
Zadeh (Fuzzy Set)	Yes	No	No	No	No	No	No
Atanassov (InFS)	Yes	Yes	No	No	No	No	No
Cuong(PicFS)	Yes	Yes	Yes	No	No	No	No
Remot et al(CFS).	Yes	No	No	Yes	Yes	No	No
Molodtsov (SS)	No	No	No	No	No	Yes	No
Maji et al.(a) (FSS)	Yes	No	No	No	No	Yes	No
Maji et al.(b) (IFSS)	Yes	Yes	No	No	No	Yes	No
Thirumunavukarasu (CFSS)	Yes	No	No	Yes	Yes	Yes	No
kumar & Bajaj (CIFSS)	Yes	Yes	No	Yes	Yes	Yes	No
Proposed (CPicFHySS)	Yes	Yes	Yes	Yes	Yes	Yes	Yes

where, Mb represents membership value, N.Mb is for non-membership degree, Neu symbolizes neutral term, Mag is magnitude factor, Phase means the phase term, P.T represents parameter Term and S.P.T symbolizes sub parameter term.

5. Conclusion

This research work proves that how CPicFHySS approach offers robust assistance for complex decision-making in uncertain situations, where conventional approaches often prove inadequate. By simultaneously evaluating both the strength of preferences (how much we favor certain options) and their potential variations over time (how these preferences might change), CPicFHySS offers a more nuanced approach to weighing multiple competing factors - particularly useful when dealing with incomplete information or shifting conditions. The framework shows promise across diverse real-world applications, from healthcare systems balancing treatment efficacy against costs to urban planners navigating uncertain population growth projections. While CPicFHySS provides a robust mathematical foundation, its true potential emerges when combined with human expertise to interpret subtle contextual factors and dynamic data systems that update assessments in real-time.

Future development should focus on making these advanced decision-support tools more intuitive and accessible, bridging the gap between theoretical sophistication and practical usability, because when facing uncertainty, what we need most aren't perfect answers but reliable ways to make better-informed choices.

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